Int. J. Multiphase Flow Vol. 6, pp. 383–384
Pergamon/Elsevier. 1980. Printed in Great Britain

LETTER TO THE EDITOR

In response to discussions of G. B. Wallis (1978) and J. A. Bouré (1979)

We appreciate the discussions of our papers on multiphase-multidomain fluid dynamics (Chao *et al.* 1978, Sha *et al.*, 1979).

Our 1978 paper pertains to multiphase flow of dispersed systems defined in the sense by Soo (1965). The need of recognizing the multi-domain characteristics (flow regimes) of other practical systems has been recently explained (Sha & Soo 1978, Soo 1979). In these publications, configuration parameters are introduced to the pressure gradient and inertial coupling terms in the momentum equations and another to account for the transfer of static enthalpy due to inertial effects in the energy equation. The numerical values of these parameters are indeed dependent upon the configuration of the multidomain flow, i.e. the flow regime. For multiphase flow of dispersed systems considered in the 1978 paper, specific numerical values may be assigned to these configuration parameters. As such, they are special cases of the more general systems. Hence, Prof. Wallis' comments are in agreement with our view as he was referring to the multidomain characteristics of the more general flow. We were not aware of Prof. Wallis' discussion when the need of including configuration parameters in the conservation equations was recognized.

The instantaneous volume-averaged linear momentum and energy equations as given by Dr. Bouré are incomplete since they do not provide for the inertial coupling effects between phases. That such effects must exist in the general multiphase-multidomain flow systems have long been pointed out by us and, interestingly enough, have also been recognized by Prof. Wallis (in his discussion of our 1978 paper).

The terms in the above papers of ours account for the transfer of waves via intertia force and pressure force across the interface of phases. It is not surprising that, when extended to include multidomain configurations, these effects enter in both the momentum and energy equations. Taking these configurations (slug and annular flows, etc.) into account and taking the example of one-dimensional flow for the sake of simplicity, these equations are (using the notations in our 1978 paper):

$$\frac{\partial}{\partial t}\rho_1 U_1 + \frac{\partial}{\partial x}\rho_1 U_1^2 = -\phi_1 \frac{\partial P}{\partial x} - (1-B_1)P\frac{\partial \phi_1}{\partial x} + C_1 \frac{\partial}{\partial x}\rho_1(U_1 - U_m)^2 + \Gamma_1 U_m + V_{12},$$

$$\frac{\partial}{\partial t}\rho_1 H_1 + \frac{\partial}{\partial x}\rho_1 U_1 H_1 = \phi_1 \frac{\partial P}{\partial t} + (1-B_1)P\frac{\partial \phi_1}{\partial t} + \frac{C_1}{2} \times \frac{\partial}{\partial x}\rho_1(U_1 - U_m)(U_1^2 - U_m^2) + C_{h1}\frac{\partial}{\partial x}\rho_1(U_1 - U_m) \times (h_1 - h_m) + \Gamma_1 H_m + U_1 V_{12} + J_{EV1} + Q_{12} + J_{E1},$$

where B_1 and C_1 are configuration parameters for the transfer of pressure and inertia forces per unit volume of phase 1; C_{h1} is that for static enthalpy h. For multiphase or dispersed flow regime, $B = 0$ and $C = 1$; and for pure stratified (or drift flux) flow, $B = 1$ and $C = 0$ as given by Dr. Bouré. When heat transfer by convection exists at the interface, $C_h = 0$. These limiting values can be rigorously shown for general motion by multiphase mechanics (Sha & Soo 1978), by volume averaging (Slattery 1967), or by time averaging (Delhaye & Archard 1976) as long as transfer of waves across the interface is accounted for. In the latter, however, ρ_1 and ϕ_1 are based on average residence time of a phase. Other terms in the above equations are J_{EV1} for the heat distributed to phase 1 due to dissipations such that $U_1 V_{12} + U_2 V_{21} = J_{EV1} + J_{EV2}$, and the heat source in phase 1 is J_{E1}.

In general, B and C are functions of configurations of the interface and the dynamic variables of the system and are only rigorously determined at present for the multiphase dispersed flow or pure stratified flow (addendum of Sha & Soo 1978). For other regimes, B and C in the range $0 < B < 1$ and $1 > C > 0$ has to be determined empirically.

The physical meaning of the imaginary characteristics given by the dynamic equations when no wave is transferred across the interface ($B = 1$, $C = 0$) is that a common set of characteristics does not exist. After all, a set of real characteristics exists in each phase. Inertial coupling physically accounts for the transfer of waves across the interface and a common set of real characteristics exists for $B < 1$ and $C > 0$.

Finally, we note that [5] given by Dr. Bouré is a special case of the more general averaging theorem due to Whitaker (1969) and Slattery (1967). This theorem relates the volume average of the divergence of a tensor and the divergence of its volume average. The restrictive consideration of flow in a pipe is unnecessary.

It is further noted that volume averaging retains different dynamic phases (Soo 1965) while time averaging reduces the whole mixture of two phases. Therefore, time and volume averaging operations are not commutative. Time averaging after volume averaging has been shown to be valid (Sha & Slattery 1979) to eliminate the high frequency fluctuations retained by instantaneous volume averages.

Dept. of Mech. Engng
University of Illinois
Urbana
IL 61801
U.S.A.

B. T. CHAO
W. T. SHA
S. L. SOO

REFERENCES

BOURÉ, J. A. 1979 On the form of the pressure terms in the momentum and energy equations of two-phase flow models. *Int. J. Multiphase Flow* **5**, 159–164.

CHAO, B. T., SHA, W. T. & SOO, S. L. 1978 *Int. J. Multiphase Flow* **4**, 219–223.

DELHAYE, J. M. & ARCHARD J. L. 1976 On the averaging operators introduced in two-phase flow modelling. Transient two-phase flow. *Proc. CSNI Specialists' Meeting*, Toronto (Edited by BANERJEE S. & WEAVER K. R.), Vol. 1, pp. 5–84. AECL.

SHA, W. T. & SLATTERY, J. C. 1979 Volume-time averaging of multi-phase flow in porous media. Tech. Memo ANL-CT-79-R-7, Argonne National Laboratory.

SHA, W. T. & SOO, S. L. 1978 Multidomain-multiphase fluid mechanics. *Int. J. Heat Mass Transfer* **21**, 1581–1595, and Addendum in press.

SHA, W. T. & SOO, S. L. 1979 *Int. J. Multiphase Flow* **5**, 153–158.

SLATTERY, J. C. 1967 General balance equation for a phase interface. *I/EC Fundls* **6**, 108–115; *ibid* **7**, 672.

SOO, S. L. 1965 Dynamics of multiphase flow systems. *I/EC Fundls* **4**, 426–433.

SOO, S. L. 1979 On the nature of equations of multiphase-multidomain mechanics. *Proc. 2nd Multiphase Flow and Heat Transfer Symp.* Hemisphere, Washington.

WALLIS, G. B. 1978 *Int. J. Multiphase Flow* **4**, 585.

WHITAKER, S. 1969 Advances in theory of fluid motion in porous media. *Ind. Engng Chem.* **61**(12), 14–28.